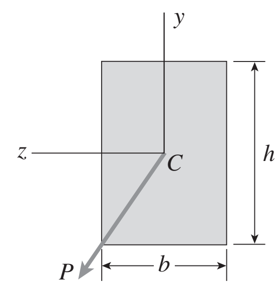
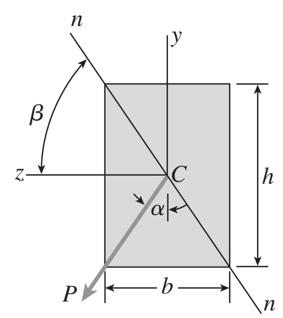
Beams with Inclined Loads

When solving the problems for Section 6.4, be sure to draw a sketch of the cross section showing the orientation of the neutral axis and the locations of the points where the stresses are being found.

Problem 6.4-1 A beam of rectangular cross section supports an inclined load P having its line of action along a diagonal of the cross section (see figure). Show that the neutral axis lies along the other diagonal.



Solution 6.4-1 Location of neutral axis



Load P acts along a diagonal.

$$\tan\alpha = \frac{b/2}{h/2} = \frac{b}{h}$$

$$I_z = \frac{bh^3}{12}$$

$$I_y = \frac{hb^2}{12}$$

$$\frac{I_z}{I_y} = \frac{h^2}{b^2}$$

See Figure 6-15b.

β = angle between the z axis and the neutral axis nn

θ = angle between the y axis and the load P

$\theta = \alpha + 180°$

$\tan\theta = \tan(\alpha + 180°) = \tan\alpha$

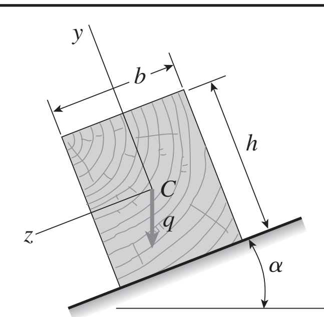
(Eq. 6-23): $$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{h^2}{b^2}\tan\theta$$

$$= \left(\frac{h^2}{b^2}\right)\left(\frac{b}{h}\right) = \frac{h}{b}$$

$\therefore$ The neutral axis lies along the other diagonal. QED ⟵

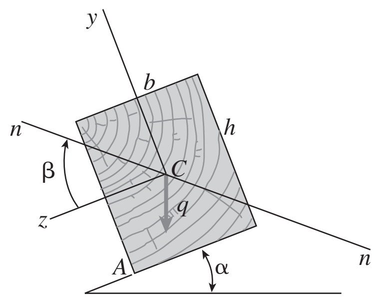
Problem 6.4-2 A wood beam of rectangular cross section (see figure) is simply supported on a span of length L. The longitudinal axis of the beam is horizontal, and the cross section is tilted at an angle α. The load on the beam is a vertical uniform load of intensity q acting through the centroid C.

Determine the orientation of the neutral axis and calculate the maximum tensile stress σ_{max} if $b = 75$ mm, $h = 150$ mm, $L = 1.5$ m, $\alpha = 30°$, and $q = 6.4$ kN/m.



Probs. 6.4-2 and 6.4-3

Solution 6.4-2 **Simple beam with inclined load**



$L = 1.5$ m $\quad q = 6.4$ kN/m $\quad b = 75$ mm
$h = 150$ mm $\quad \alpha = 30°$

Bending moments

$$M_y = \frac{q_z L^2}{8} = \frac{q(\sin\alpha)L^2}{8} = 900 \text{ N} \cdot \text{m}$$

$$M_z = \frac{q_y L^2}{8} = \frac{q(\cos\alpha)L^2}{8} = 1559 \text{ N} \cdot \text{m}$$

Moments of inertia

$$I_y = \frac{hb^3}{12} = 5{,}273 \times 10^3 \text{ mm}^4$$

$$I_z = \frac{bh^3}{12} = 21{,}094 \times 10^3 \text{ mm}^4$$

Neutral axis nn (Eq. 6-23)

$$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{I_z}{I_y}\tan\alpha = \left(\frac{h}{b}\right)^2 \tan\alpha = 4\tan 30° = 2.3094$$

$$\beta = 66.6° \quad \longleftarrow$$

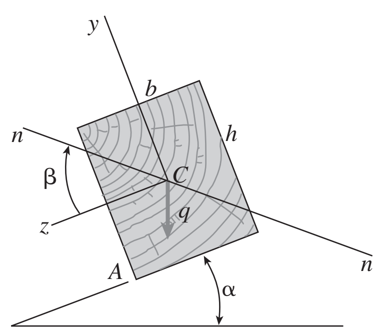
Maximum tensile stress (at point A) (Eq. 6-18)

$$\sigma_{\max} = \frac{M_y(b/2)}{I_y} - \frac{M_z(-h/2)}{I_z} = \frac{(900 \text{ N}\cdot\text{m})(37.5 \text{ mm})}{5273 \times 10^3 \text{ mm}^4} + \frac{(1559 \text{ N}\cdot\text{m})(75 \text{ mm})}{21{,}094 \times 10^3 \text{ mm}^4}$$

$$\sigma_{\max} = 11.9 \text{ MPa} \quad \longleftarrow$$

Problem 6.4-3 Solve the preceding problem for the following data: $b = 6$ in., $h = 8$ in., $L = 8.0$ ft, $\tan\alpha = 1/3$, and $q = 375$ lb/ft.

Solution 6.4-3 **Simple beam with inclined load**



$L = 8.0$ ft $\quad q = 375$ lb/ft $\quad b = 6$ in. $\quad h = 8$ in.

$\tan\alpha = 1/3 \quad \sin\alpha = \frac{1}{\sqrt{10}} \quad \cos\alpha = \frac{3}{\sqrt{10}}$

Bending moments

$$M_y = \frac{q_z L^2}{8} = \frac{q(\sin\alpha)L^2}{8} = 11{,}380 \text{ lb-in.}$$

$$M_z = \frac{q_y L^2}{8} = \frac{q(\cos\alpha)L^2}{8} = 34{,}150 \text{ lb-in.}$$

Moments of inertia

$$I_y = \frac{hb^3}{12} = 144 \text{ in.}^4 \qquad I_z = \frac{bh^3}{12} = 256 \text{ in.}^4$$

Neutral axis nn (Eq. 6-23)

$$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{I_z}{I_y}\tan\alpha = \left(\frac{h}{b}\right)^2 \tan\alpha = 0.5926 \qquad \beta = 30.7° \quad \longleftarrow$$

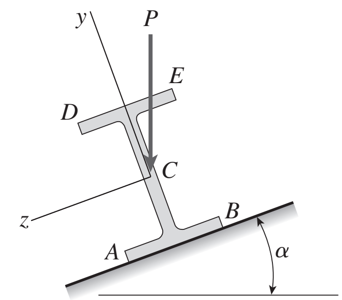
Maximum tensile stress (at point A) (Eq. 6-18)

$$\sigma_{\max} = \frac{M_y(b/2)}{I_y} - \frac{M_z(-h/2)}{I_z} = 771 \text{ psi} \quad \longleftarrow$$

Problem 6.4-4 A simply supported wide-flange beam of span length L carries a vertical concentrated load P acting through the centroid C at the midpoint of the span (see figure). The beam is attached to supports inclined at an angle α to the horizontal.

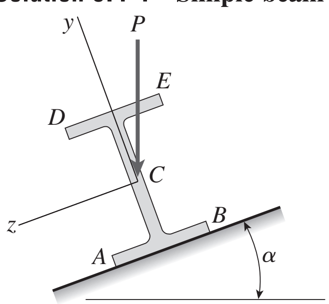
Determine the orientation of the neutral axis and calculate the maximum stresses at the outside corners of the cross section (points A, B, D, and E) due to the load P.

Data for the beam are as follows: W 10×30 section, $L = 10.0$ ft, $P = 4.25$ k, and $\alpha = 26.57°$. (*Note:* See Table E-1 of Appendix E for the dimensions and properties of the beam.)



Probs. 6.4-4 and 6.4-5

Solution 6.4-4 Simple beam with inclined load



$L = 10.0$ ft
$P = 4.25$ k
$\alpha = 26.57°$

Wide-flange beam:
W 10×30 $I_y = 16.7$ in.^{4} $I_z = 170$ in.^{4}
$d = 10.47$ in. $b = 5.810$ in.

BENDING MOMENTS

$$M_y = \frac{P(\sin\alpha)L}{4} = 57{,}030 \text{ lb-in.}$$

$$M_z = \frac{P(\cos\alpha)L}{4} = 114{,}030 \text{ lb-in.}$$

NEUTRAL AXIS nn (EQ. 6-23)

$$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{I_z}{I_y}\tan\alpha = 5.0909$$

$$\beta = 78.89° \longleftarrow$$

BENDING STRESSES (EQ. 6-18)

$$\sigma_x = \frac{M_y z}{I_y} - \frac{M_z y}{I_z} = \frac{57{,}030 \text{ lb-in.}}{16.7 \text{ in.}^4}(z) - \frac{114{,}030 \text{ lb-in.}}{170 \text{ in.}^4}(y)$$

Point A: $z_A = \frac{b}{2} = 2.905$ in.

$$y_A = -\frac{d}{2} = -5.235 \text{ in.}$$

$$\sigma_A = -\sigma_E = 13{,}430 \text{ psi} \longleftarrow$$

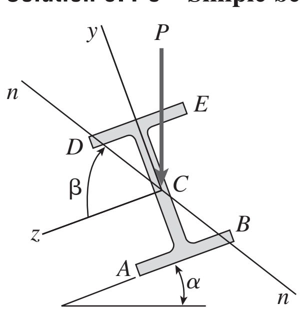
Point B: $z_B = -\frac{b}{2} = -2.905$ in.

$$y_B = -\frac{d}{2} = -5.235 \text{ in.}$$

$$\sigma_B = -\sigma_D = -6410 \text{ psi} \longleftarrow$$

Problem 6.4-5 Solve the preceding problem using the following data: W 8×21 section, $L = 76$ in., $P = 4.8$ k, and $\alpha = 20°$.

Solution 6.4-5 Simple beam with inclined load



$L = 76$ in.
$P = 4.8$ k
$\alpha = 20°$

Wide-flange beam:
W 8×21 $I_y = 9.77$ in.^{4} $I_z = 75.3$ in.^{4}
$d = 8.28$ in. $b = 5.270$ in.

BENDING MOMENTS

$$M_y = \frac{P(\sin\alpha)L}{4} = 31{,}190 \text{ lb-in.}$$

$$M_z = \frac{P(\cos\alpha)L}{4} = 85{,}700 \text{ lb-in.}$$

NEUTRAL AXIS nn (EQ. 6-23)

$$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{I_z}{I_y}\tan\alpha = 2.8052$$

$$\beta = 70.38° \longleftarrow$$

BENDING STRESSES (EQ. 6-18)

$$\sigma_x = \frac{M_y z}{I_y} - \frac{M_z y}{I_z} = \frac{31{,}190 \text{ lb-in.}}{9.77 \text{ in.}^4}(z) - \frac{85{,}700 \text{ lb-in.}}{75.3 \text{ in.}^4}(y)$$

Point A: $z_A = \frac{b}{2} = 2.635$ in.

$$y_A = -\frac{d}{2} = -4.140 \text{ in.}$$

$$\sigma_A = -\sigma_E = 13{,}120 \text{ psi} \longleftarrow$$

Point B: $z_B = -\frac{b}{2} = -2.635$ in.

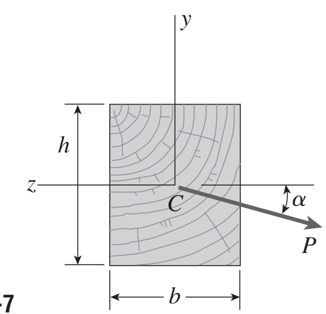
$$y_B = -\frac{d}{2} = -4.140 \text{ in.}$$

$$\sigma_B = -\sigma_D = -3700 \text{ psi} \longleftarrow$$

Problem 6.4-6 A wood cantilever beam of rectangular cross section and length L supports an inclined load P at its free end (see figure).

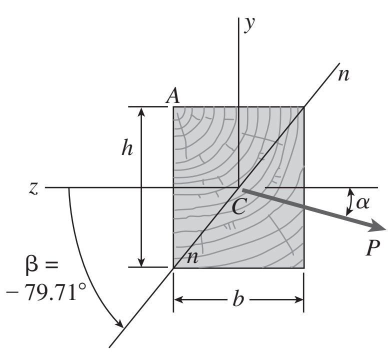
Determine the orientation of the neutral axis and calculate the maximum tensile stress σ_{max} due to the load P.

Data for the beam are as follows: $b = 75$ mm, $h = 150$ mm, $L = 1.8$ m, $P = 625$ N, and $\alpha = 36°$.



Probs. 6.4-6 and 6.4-7

Solution 6.4-6 Cantilever beam with inclined load



$P = 625$ N $\quad L = 1.8$ m $\quad \alpha = 36°$

$b = 75$ mm $\quad h = 150$ mm

$$I_y = \frac{bh^3}{12} = 5.273 \times 10^6 \text{ mm}^4$$

$$I_a = \frac{bh^3}{12} = 21.094 \times 10^6 \text{ mm}^4$$

BENDING MOMENTS

$$M_y = (P \cos \alpha)L = 910.1 \text{ N} \cdot \text{m}$$

$$M_z = -(P \sin \alpha)L = -661.3 \text{ N} \cdot \text{m}$$

NEUTRAL AXIS nn (EQ. 6-23)

$$\tan \beta = \frac{I_z}{I_y} \tan \theta \qquad \theta = \alpha + 90°$$

$$\tan \beta = \frac{21.094}{5.273} \tan(36° + 90°) = -5.5060$$

$$\beta = -79.71° \longleftarrow$$

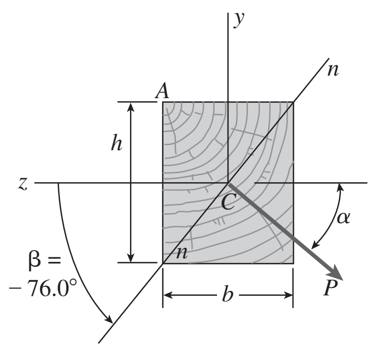
MAXIMUM TENSILE STRESS (POINT A) (EQ. 6-18)

$z_A = b/2 = 37.5$ mm $\qquad y_A = h/2 = 75$ mm

$$\sigma_{max} = \sigma_A = \frac{M_y z_A}{I_y} - \frac{M_z y_A}{I_z} = 8.82 \text{ MPa} \longleftarrow$$

Problem 6.4-7 Solve the preceding problem for a cantilever beam with data as follows: $b = 4$ in., $h = 8$ in., $L = 7.5$ ft, $P = 320$ lb, and $\alpha = 45°$.

Solution 6.4-7 **Cantilever beam with inclined load**



$P = 320$ lb $\quad L = 7.5$ ft $= 90$ in.
$\alpha = 45°$ $\quad b = 4$ in. $\quad h = 8$ in.

$$I_y = \frac{hb^3}{12} = 42.667 \text{ in.}^4$$

$$I_z = \frac{bh^3}{12} = 170.67 \text{ in.}^4$$

Bending moments

$M_y = (P \cos \alpha)L = 20{,}365$ lb-in.
$M_z = -(P \sin \alpha)L = -20{,}365$ lb-in.

Neutral axis *nn* (Eq. 6-23)

$$\tan\beta = \frac{I_z}{I_y}\tan\theta \qquad \theta = \alpha + 90°$$

$$\tan\beta = \frac{170.67}{42.667}\tan(45° + 90°) = -4.000$$

$$\beta = -75.96° \quad \longleftarrow$$

Maximum tensile stress (point *A*) (Eq. 6-18)

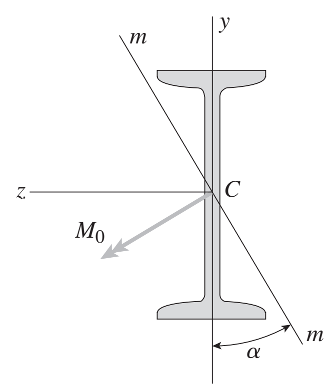
$z_A = b/2 = 2$ in. $\quad y_A = h/2 = 4$ m

$$\sigma_{max} = \sigma_A = \frac{M_y z_A}{I_y} - \frac{M_z y_A}{I_z} = 1430 \text{ psi} \quad \longleftarrow$$

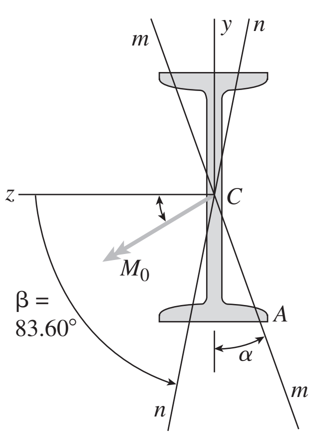
Problem 6.4-8 A steel beam of I-section (see figure) is simply supported at the ends. Two equal and oppositely directed bending moments M_0 act at the ends of the beam, so that the beam is in pure bending. The moments act in plane *mm*, which is oriented at an angle α to the *xy* plane.

Determine the orientation of the neutral axis and calculate the maximum tensile stress σ_{max} due to the moments M_0.

Data for the beam are as follows: S 8 × 18.4 section, $M_0 = 30$ k-in., and $\alpha = 30°$. (*Note:* See Table E-2 of Appendix E for the dimensions and properties of the beam.)



Solution 6.4-8 **Beam in pure bending**



$M_0 = 30$ k-in. $= 30{,}000$ lb-in.
$\alpha = 30°$ $\quad$ S 8 × 18.4 $\quad I_y = 3.73$ in.^{4}
$I_z = 57.6$ in.^{4} $\quad d = 8.00$ in. $\quad b = 4.001$ in.
$M_y = -M_0 \sin \alpha = -15{,}000$ lb-in.
$M_z = M_0 \cos \alpha = 25{,}980$ lb-in.

Neutral axis *nn* (Eq. 6-23)

$\theta = -\alpha = -30°$ (see Fig. 6-15)

$$\tan \beta = \frac{I_z}{I_y}\tan\theta = \frac{57.6}{3.73}\tan(-30°) = -8.9157$$

$$\beta = -83.60° \quad \longleftarrow$$

Maximum tensile stress (point *A*) (Eq. 6-18)

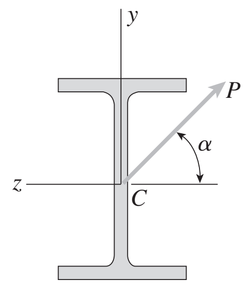
$z_A = -b/2 = -2.000$ in.
$y_A = -d/2 = -4.000$ in.

$$\sigma_{max} = \frac{M_y z_A}{I_y} - \frac{M_z y_a}{I_z} = 9850 \text{ psi} \quad \longleftarrow$$

Problem 6.4-9 A cantilever beam of wide-flange cross section and length L supports an inclined load P at its free end (see figure).

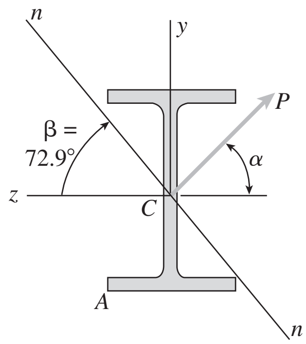
Determine the orientation of the neutral axis and calculate the maximum tensile stress σ_{max} due to the load P.

Data for the beam are as follows: W 10 × 45 section, $L = 8.0$ ft, $P = 1.5$ k, and $\alpha = 55°$. (*Note:* See Table E-1 of Appendix E for the dimensions and properties of the beam.)



Probs. 6.4-9 and 6.4-10

Solution 6.4-9 Cantilever beam with inclined load



$P = 1.5 \text{ k} = 1500 \text{ lb}$
$L = 8.0 \text{ ft} = 96 \text{ in.}$
$\alpha = 55°$
W 10 × 45
$I_y = 53.4 \text{ in.}^4 \quad I_z = 248 \text{ in.}^4$
$d = 10.10 \text{ in.} \quad b = 8.02 \text{ in.}$

BENDING MOMENTS

$M_y = (P \cos \alpha)L = 82{,}595 \text{ lb-in.}$
$M_z = (P \sin \alpha)L = 117{,}960 \text{ lb-in.}$

NEUTRAL AXIS nn (EQ. 6-23)

$\theta = 90° - \alpha = 35°$ (see Fig. 6-15)

$$\tan \beta = \frac{I_z}{I_y} \tan \theta = \frac{248}{53.4} \tan 35° = 3.2519$$

$\beta = 72.91°$ ⟵

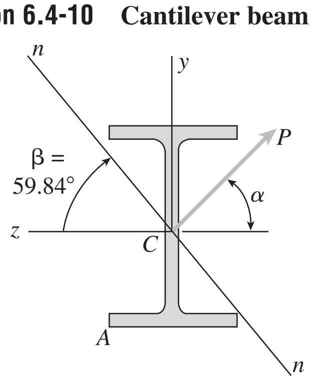
MAXIMUM TENSILE STRESS (POINT A) (EQ. 6-18)

$z_A = b/2 = 4.01 \text{ in.}$
$y_A = -d/2 = -5.05 \text{ in.}$

$$\sigma_{max} = \sigma_A = \frac{M_y z_A}{I_y} - \frac{M_z y_A}{I_z} = 8600 \text{ psi}$$ ⟵

Problem 6.4-10 Solve the preceding problem using the following data: W 8 × 35 section, $L = 5.0$ ft, $P = 2.4$ k, and $\alpha = 60°$.

Solution 6.4-10 Cantilever beam with inclined load



$P = 2.4 \text{ k} = 2400 \text{ lb}$
$L = 5.0 \text{ ft} = 60 \text{ in.}$
$\alpha = 60°$
W 8 × 35
$I_y = 42.6 \text{ in.}^4 \quad I_z = 127 \text{ in.}^4$
$d = 8.12 \text{ in.} \quad b = 8.020 \text{ in.}$

BENDING MOMENTS

$M_y = (P \cos \alpha)L = 72{,}000 \text{ lb-in.}$
$M_z = (P \sin \alpha)L = 124{,}710 \text{ lb-in.}$

NEUTRAL AXIS nn (EQ. 6-23)

$\theta = 90° - \alpha = 30°$ (see Fig. 6-15)

$$\tan \beta = \frac{I_z}{I_y} \tan \theta = \frac{127}{426} \tan 30° = 1.7212$$

$\beta = 59.84°$ ⟵

MAXIMUM TENSILE STRESS (POINT A) (EQ. 6-18)

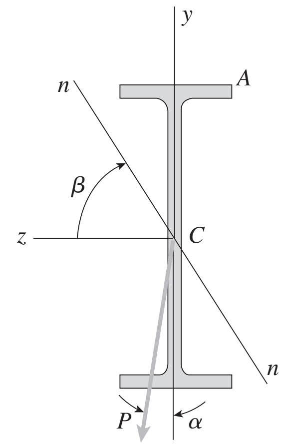
$z_A = b/2 = 4.01 \text{ in.}$
$y_A = -d/2 = -4.06 \text{ in.}$

$$\sigma_{max} = \sigma_A = \frac{M_y z_A}{I_y} - \frac{M_z y_A}{I_z} = 10{,}760 \text{ psi}$$ ⟵

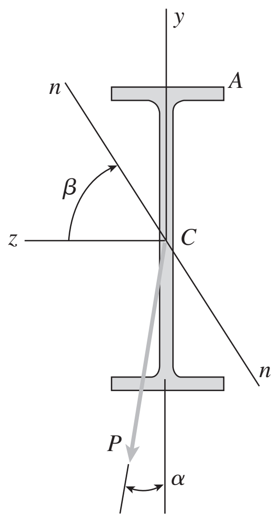
Problem 6.4-11 A cantilever beam of W 12 × 14 section and length $L = 9$ ft supports a slightly inclined load $P = 500$ lb at the free end (see figure).

(a) Plot a graph of the stress σ_A at point A as a function of the angle of inclination α.

(b) Plot a graph of the angle β, which locates the neutral axis nn, as a function of the angle α. (When plotting the graphs, let α vary from 0 to 10°.) (*Note:* See Table E-1 of Appendix *E* for the dimensions and properties of the beam.)



Solution 6.4-11 **Cantilever beam with inclined load**



$P = 500$ lb $\quad L = 9$ ft $= 108$ in. $\quad$ W 12 × 14

$I_y = 2.36$ in.^{4} $\quad I_z = 88.6$ in.^{4}

$d = 11.91$ in. $\quad b = 3.970$ in.

BENDING MOMENTS

$M_y = -(P \sin \alpha)L = -54{,}000 \sin \alpha$

$M_z = -(P \cos \alpha)L = -54{,}000 \cos \alpha$

(a) STRESS AT POINT A (EQ. 6-18)

$z_A = -b/2 = -1.985$ in.

$y_A = d/2 = 5.955$ in.

$$\sigma_A = \frac{M_y z_A}{I_y} - \frac{M_z y_A}{I_z} = 45{,}420 \sin \alpha + 3629 \cos \alpha \text{ (psi)} \longleftarrow$$

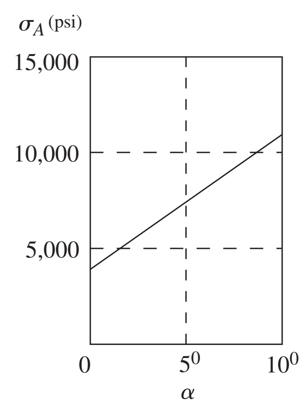
(b) NEUTRAL AXIS nn (EQ. 6-23)

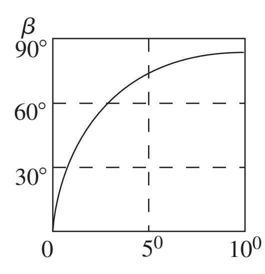
$\theta = 180° + \alpha \quad$ (see Fig. 6-15)

$$\tan \beta = \frac{I_z}{I_y} \tan \theta = \frac{I_z}{I_y} \tan(180° + \alpha)$$

$$= \frac{88.6}{2.36} \tan(180° + \alpha) = 37.54 \tan \alpha$$

$$\beta = \arctan(37.54 \tan \alpha) \longleftarrow$$





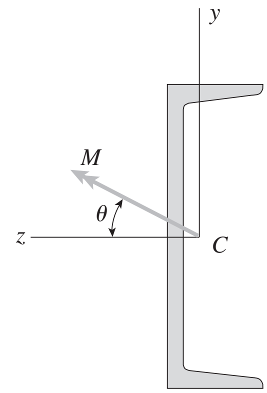
Bending of Unsymmetric Beams

When solving the problems for Section 6.5, be sure to draw a sketch of the cross section showing the orientation of the neutral axis and the locations of the points where the stresses are being found.

Problem 6.5-1 A beam of channel section is subjected to a bending moment M having its vector at an angle θ to the z axis (see figure).

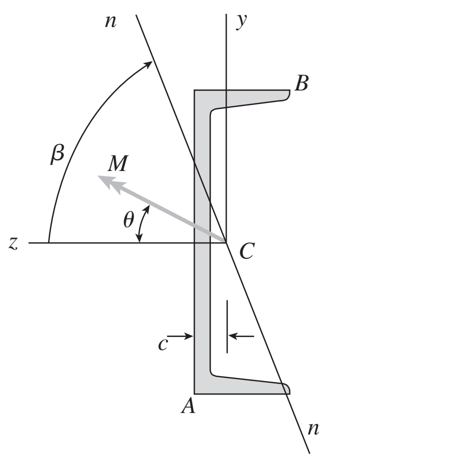
Determine the orientation of the neutral axis and calculate the maximum tensile stress σ_t and maximum compressive stress σ_c in the beam.

Use the following data: C 8 × 11.5 section, $M = 20$ k-in., $\tan \theta = 1/3$. (*Note:* See Table E-3 of Appendix E for the dimensions and properties of the channel section.)



Probs. 6.5-1 and 6.5-2

Solution 6.5-1 **Channel section**



$M = 20$ k-in. $\tan \theta = 1/3$ $\theta = 18.435°$
C 8 × 11.5
$c = 0.571$ in. $I_y = 1.32$ in.^{4} $I_z = 32.6$ in.^{4}
$d = 8.00$ in. $b = 2.260$ in.

NEUTRAL AXIS *nn* (EQ. 6-40)

$$\tan \beta = \frac{I_z}{I_y} \tan \theta = \frac{32.6}{1.32}(1/3) = 8.2323$$

$$\beta = 83.07° \longleftarrow$$

MAXIMUM TENSILE STRESS (POINT *A*) (EQ. 6-38)

$z_A = c = 0.571$ in. $y_A = -d/2 = -4.00$ in.

$$\sigma_t = \sigma_A = \frac{(M \sin \theta) z_A}{I_y} - \frac{(M \cos \theta) y_A}{I_z}$$

$$= 5060 \text{ psi} \longleftarrow$$

MAXIMUM COMPRESSIVE STRESS (POINT *B*) (EQ. 6-38)

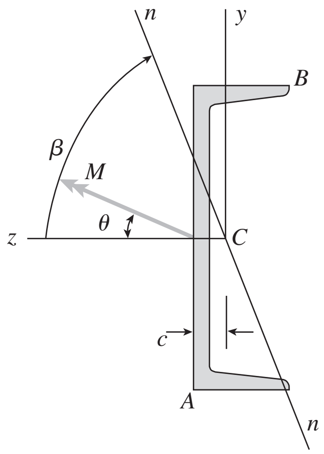
$z_B = -(b - c) = -(2.260 - 0.571) = -1.689$ in.
$y_z = d/2 = 4.00$ in.

$$\sigma_c = \sigma_B = \frac{(M \sin \theta) z_B}{I_y} - \frac{(M \cos \theta) y_B}{I_z}$$

$$= -10{,}420 \text{ psi} \longleftarrow$$

Problem 6.5-2 Solve the preceding problem for a C 6 × 13 channel section with $M = 5.0$ k-in. and $\theta = 15°$.

Solution 6.5-2 **Channel section**



$M = 5.0$ k-in. $\theta = 35°$ C 6 × 13
$c = 0.514$ in. $I_y = 1.05$ in.^{4} $I_z = 17.4$ in.^{4}
$d = 6.00$ in. $b = 2.157$ in.

NEUTRAL AXIS *nn* (EQ. 6-40)

$$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{17.4}{1.05}\tan 15° = 4.4403$$

$$\beta = 77.31° \longleftarrow$$

MAXIMUM TENSILE STRESS (POINT *A*) (EQ. 6-38)

$z_A = c = 0.514$ in. $y_A = -d/2 = -3.00$ in.

$$\sigma_t = \sigma_A = \frac{(M\sin\theta)z_A}{I_y} - \frac{(M\cos\theta)y_A}{I_z}$$
$$= 1470 \text{ psi} \longleftarrow$$

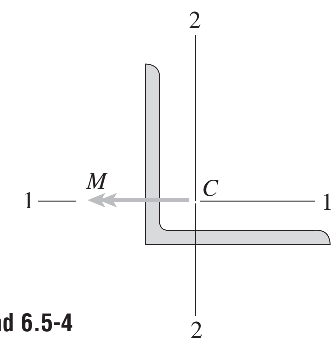
MAXIMUM COMPRESSIVE STRESS (POINT *B*) (EQ. 6-38)

$z_B = -(b - c) = -(2.157 - 0.514) = -1.643$ in.
$y_B = d/2 = 3.00$ in.

$$\sigma_c = \sigma_B = \frac{(M\sin\theta)z_B}{I_y} - \frac{(M\cos\theta)y_B}{I_z}$$
$$= -2860 \text{ psi} \longleftarrow$$

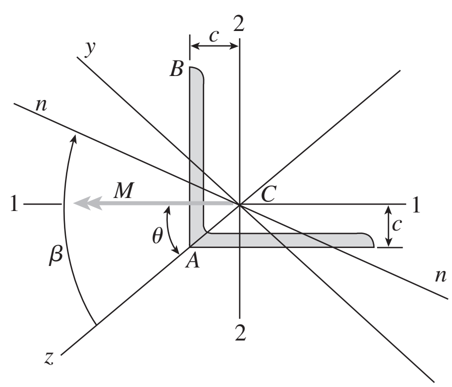
Problem 6.5-3 An angle section with equal legs is subjected to a bending moment M having its vector directed along the 1-1 axis, as shown in the figure.

Determine the orientation of the neutral axis and calculate the maximum tensile stress σ_t and maximum compressive stress σ_c if the angle is an L 6 × 6 × 3/4 section and $M = 20$ k-in. (*Note:* See Table E-4 of Appendix E for the dimensions and properties of the angle section.)



Probs. 6.5-3 and 6.5-4

Solution 6.5-3 **Angle section with equal legs**



$M = 20$ k-in. L 6 × 6 × 3/4 in.
$h = b = 6$ in. $c = 1.78$ in.
$I_1 = I_2 = 28.2$ in.^{4}
$\theta = 45°$ $A = 8.44$ in.^{2} $r_{min} = 1.17$ in.
$I_y = Ar^2_{min} = 11.55$ in.^{4}
$I_z = I_1 + I_2 - I_y = 44.85$ in.^{4}

NEUTRAL AXIS *nn* (EQ. 6-40)

$$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{44.85}{11.55}\tan 45° = 3.8831$$

$$\beta = 75.56° \longleftarrow$$

MAXIMUM TENSILE STRESS (POINT *A*) (EQ. 6-38)

$z_A = c\sqrt{2} = 2.517$ in. $y_A = 0$

$$\sigma_t = \sigma_A = \frac{(M\sin\theta)z_A}{I_y} - \frac{(M\cos\theta)y_A}{I_z} = 3080 \text{ psi} \longleftarrow$$

MAXIMUM COMPRESSIVE STRESS (POINT *B*) (EQ. 6-38)

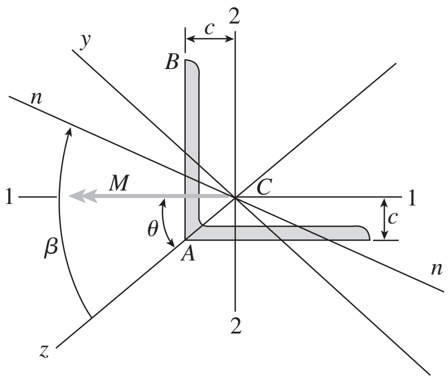
$z_B = c\sqrt{2} - h/\sqrt{2} = -1.725$ in.

$y_B = h/\sqrt{2} = 4.243$ in.

$$\sigma_c = \sigma_B = \frac{(M\sin\theta)z_B}{I_y} - \frac{(M\cos\theta)y_B}{I_z} = -3450 \text{ psi} \longleftarrow$$

Problem 6.5-4 Solve the preceding problem for an L 4 × 4 × 1/2 angle section with $M = 6.0$ k-in.

Solution 6.5-4 **Angle section with equal legs**



$M = 6.0$ k-in. L 4 × 4 × 1/2 in.

$h = b = 4.0$ in.

$c = 1.18$ in. $I_1 = I_2 = 5.56$ in.^{4}

$\theta = 45°$ $A = 3.75$ in.^{2} $r_{\min} = 0.782$ in.

$I_y = Ar_{\min}^2 = 2.293$in.^{4}

$I_z = I_1 + I_2 - I_y = 8.827$ in.^{4}

NEUTRAL AXIS *nn* (EQ. 6-40)

$$\tan\beta = \frac{I_z}{I_y}\tan\theta = \frac{8.827}{2.293}\tan 45° = 3.8495$$

$$\beta = 75.44° \longleftarrow$$

MAXIMUM TENSILE STRESS (POINT *A*) (EQ. 6-38)

$$z_A = c\sqrt{2} = 1.669 \text{ in.} \qquad y_A = 0$$

$$\sigma_t = \sigma_A = \frac{(M\sin\theta)z_A}{I_y} - \frac{(M\cos\theta)y_A}{I_z}$$

$$= 3090 \text{ psi} \longleftarrow$$

MAXIMUM COMPRESSIVE STRESS (POINT *B*) (EQ. 6-38)

$$z_B = c\sqrt{2} - h/\sqrt{2} = -1.160 \text{ in.}$$

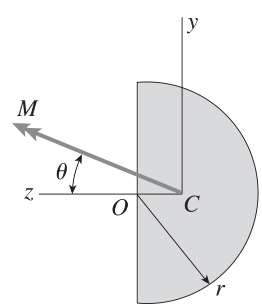
$$y_B = h/\sqrt{2} = 2.828 \text{ in.}$$

$$\sigma_c = \sigma_B = \frac{(M\sin\theta)z_B}{I_y} - \frac{(M\cos\theta)y_B}{I_z}$$

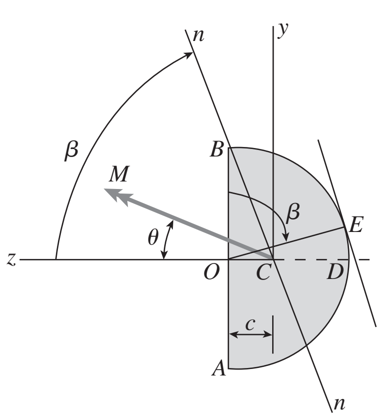
$$= -3510 \text{ psi} \longleftarrow$$

Problem 6.5-5 A beam of semicircular cross section of radius r is subjected to a bending moment M having its vector at an angle θ to the z axis (see figure).

Derive formulas for the maximum tensile stress σ_t and the maximum compressive stress σ_c in the beam for $\theta = 0$, 45°, and 90°. (*Note*: Express the results in the form $\alpha M/r^3$, where α is a numerical value.)



Solution 6.5-5 **Semicircle**



r = radius

$$c = \frac{4r}{3\sigma} = 0.42441r$$

$$I_y = \frac{(9\sigma^2 - 64)}{72\sigma}r^4$$

$$= 0.109757r^4$$

$$I_z = \frac{\pi r^4}{8}$$

σ_t = maximum tensile stress
σ_c = maximum compressive stress

FOR $\theta = 0°$: $\sigma_t = \sigma_A = \dfrac{Mr}{I_z} = \dfrac{8M}{\pi r^3}$

$$= 2.546 \frac{M}{r^3} \longleftarrow$$

$$\sigma_c = \sigma_B = -\sigma_A = -\frac{8M}{\pi r^3}$$

$$= -2.546 \frac{M}{r^3} \longleftarrow$$

FOR $\theta = 90°$: $\sigma_t = \sigma_0 = \dfrac{Mc}{I_y}$

$$= 3.867 \frac{M}{r^3} \longleftarrow$$

$$\sigma_c = \sigma_D = \frac{M(r-c)}{I_y}$$

$$= -5.244 \frac{M}{r^3} \longleftarrow$$

FOR $\theta = 45°$: Eq. (6-40): $\tan\beta = \dfrac{I_z}{I_y}\tan\theta$

$$\tan\beta = \frac{9\pi^2}{9\pi^2 - 64}(1) = 3.577897$$

$\beta = 74.3847°$
$90° - \beta = 15.6153°$

MAXIMUM TENSILE STRESS for $\theta = 45°$ occurs at point A.

$z_A = c = 0.42441r \qquad y_A = -r$

From (Eq. 6-38):

$$\sigma_t = \sigma_A = \frac{(M\sin\theta)z_A}{I_y} - \frac{(M\cos\theta)y_A}{I_z}$$

$$= 4.535 \frac{M}{r^3} \longleftarrow$$

MAXIMUM COMPRESSIVE STRESS for $\theta = 45°$ occurs at point E, where the tangent to the circle is parallel to the neutral axis nn.
$z_E = c - r\cos(90° - \beta) = -0.53868r$
$y_E = r\sin(90° - \beta) = 0.26918r$

From (Eq. 6-38):

$$\sigma_c = \sigma_E = \frac{(M\sin\theta)z_E}{I_y} - \frac{(M\cos\theta)y_E}{I_z}$$

$$= -3.955 \frac{M}{r^3} \longleftarrow$$

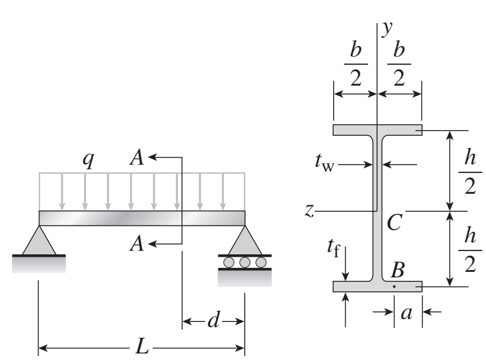
Shear Stresses in Wide-Flange Beams

When solving the problems for Section 6.8, assume that the cross sections are thin-walled. Use centerline dimensions for all calculations and derivations, unless otherwise specified

Problem 6.8-1 A simple beam of wide-flange cross section supports a uniform load of intensity $q = 3.0$ k/ft on a span of length $L = 10$ ft (see figure on the next page). The dimensions of the cross section are $h = 10.5$ in., $b = 7$ in., and $t_f = t_w = 0.4$ in.

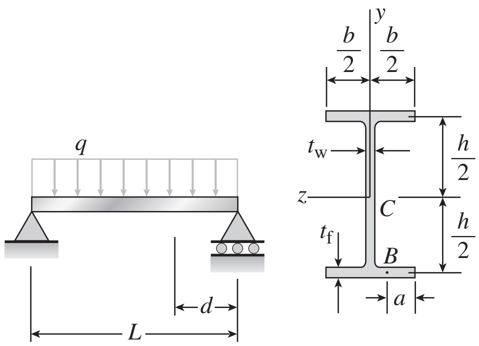
(a) Calculate the maximum shear stress τ_{max} on cross section A-A located at distance $d = 2.0$ ft from the end of the beam.

(b) Calculate the shear stress τ_B at point B on the cross section. Point B is located at a distance $a = 2.0$ in. from the edge of the lower flange.



Probs. 6.8-1 and 6.8-2

Solution 6.8-1 **Simple beam with wide-flange cross section**



CROSS SECTION

$h = 10.5$ in. $b = 7$ in. $t_f = 0.4$ in.
$t_w = 0.4$ in.

Eq. (6-57): $I_z = \dfrac{t_w h^3}{12} + \dfrac{bt_f h^2}{2} = 192.94 \text{ in.}^4$

(a) MAXIMUM SHEAR STRESS (EQ. 6-54)

$$\tau_{max} = \left(\frac{bt_f}{t_w} + \frac{h}{4}\right)\frac{Vh}{2I_z} = 2360 \text{ psi} \quad \longleftarrow$$

SIMPLE BEAM

$q = 3.0$ k/ft $L = 10$ ft $R = \dfrac{qL}{2} = 15.0$ k

$d = 2.0$ ft $V = |R - qd| = 9.0$ k

(b) SHEAR STRESS AT POINT B

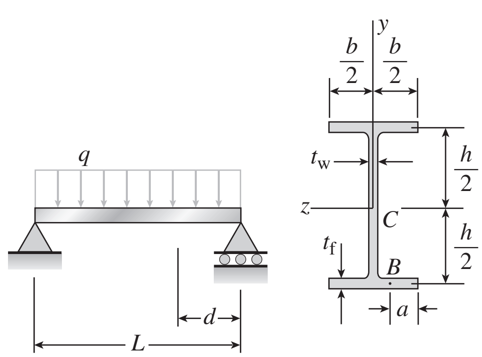
$a = 2.0$ in. $b/2 = 3.5$ in.

Eq. (6-49): $\tau_1 = \dfrac{bhV}{4I_z} = 857.1$ psi

$$\tau_B = \frac{a}{b/2}(\tau_1) = 490 \text{ psi} \quad \longleftarrow$$

Problem 6.8-2 Solve the preceding problem for the following data: $L = 3$ m, $q = 40$ kN/m, $h = 260$ mm, $b = 170$ mm, $t_f = 12$ mm, $t_w = 10$ mm, $d = 0.6$ m, and $a = 60$ mm.

Solution 6.8-2 **Simple beam with wide-flange cross section**



SIMPLE BEAM

$q = 40$ kN/m $L = 3$ m $R = \dfrac{qL}{2} = 60$ kN

$d = 0.6$ m $V = |R - qd| = 36$ kN

CROSS SECTION (FIG. 6-34)

$h = 260$ mm $b = 170$ mm $t_f = 12$ mm
$t_w = 10$ mm

Eq. (6-57): $I_z = \dfrac{t_w h^3}{12} + \dfrac{bt_f h^2}{2}$
$= 83.599 \times 10^6 \text{ mm}^4$

(a) MAXIMUM SHEAR STRESS (EQ. 6-54)

$$\tau_{max} = \left(\frac{bt_f}{t_w} + \frac{h}{4}\right)\frac{Vh}{2I_z} = 15.1 \text{ MPa} \quad \longleftarrow$$

(b) SHEAR STRESS AT POINT B

$a = 60$ mm $b/2 = 85$ mm

Eq. (6-49): $\tau_1 = \dfrac{bhV}{4I_z} = 4.758$ MPa

$$\tau_B = \frac{a}{b/2}(\tau_1) = 3.4 \text{ MPa} \quad \longleftarrow$$